ON GLOBAL STRUCTURE OF HADRONIC TOTAL CROSS SECTIONS

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Simple theoretical formula describing the global structure of pp and $p\bar{p}$ total cross sections in the whole range of energies available up today has been derived. The fit to the experimental data with the formula has been made. It is shown that there is a very good correspondence of the theoretical formula to the existing experimental data.

I. INTRODUCTION

It is a well known fact that at energies above $\sqrt{s} \sim 20\,GeV$ all hadronic total cross sections rise with the growth of energy. In 1970 the experiments at the Serpukhov accelerator revealed that the K^+p total cross section increased with energy [1]. Increase of the pp total cross section has been discovered at the CERN ISR [2] and then the effect of rising total cross sections was confirmed at the Fermilab accelerator [3].

Although nowadays we have in the framework of local quantum field theory a gauge model of strong interactions formulated in terms of the known QCD Lagrangian its relations to the so called "soft" (interactions at large distances) hadronic physics are far from desired. Obviously the understanding of this physics is of high interest because it has intrinsically fundamental nature. In spite of more than 25 years after the formulation of QCD, we cannot still obtain from the QCD Lagrangian the answer to the question why all hadronic total cross sections grow with energy. We cannot predict total cross sections in an absolute way starting from the fundamental QCD Lagrangian as well mainly because it is not a perturbative problem. We know e.g. that nonperturbative contributions to the gluon propagator influence the behaviour of "soft" hadronic processes and the knowledge of the infrared behaviour of QCD is certainly needed to describe the "soft" hadronic physics in the framework of QCD. Unfortunately, today we don't know the whole picture of the infrared behaviour of QCD, we have some fragments of this picture though (see e.g. Ref. [4]).

At the same time it is more or less clear now that the rise of the total cross sections is just the shadow (not antishadow!) of particle production.

Through the optical theorem the total cross section is related to the imaginary part of the elastic scattering amplitude in the forward direction. That is why the theoretical understanding of elastic scattering is of the fundamental importance. From the unitarity relation it follows also that the imaginary part of the elastic scattering amplitude contains the contribution of all possible inelastic channels in two-particle interaction. It is clear therefore that we cannot understand elastic scattering without understanding inelastic interaction.

A variety of different approaches to the dynamics of "soft" hadronic processes can conditionally be divided into two groups corresponding to different forms of strong interaction dynamics: t-channel form and s-channel one.

In the framework of t-channel form of the dynamics the popular Regge phenomenology represents elastic and inelastic diffractive scattering by the exchange of the Pomeron, a color singlet Reggeon with quantum numbers of the vacuum. It should be noted, that the definition of the Pomeron as Reggeon with the highest Regge trajectory $\alpha_P(t)$, that carries the quantum numbers of the vacuum, is not only one.[1] There are many other definitions of the Pomeron: Pomeron is a gluon "ladder" [5]; Pomeron is a bound state of two reggezied gluons – BFKL-Pomeron [6]; soft and hard Pomerons [7,8], etc. This leapfrog is because of the exact nature of the Pomeron and its detailed substructure remains such as that anyone doesn't know what it is. The difficulty of establishing the true nature of the Pomeron in QCD is almost obviously related to the calculations of non-perturbative gluon exchange.

Nevertheless in the near past simple formulae of the Regge phenomenology provided good parameterization of experimental data on "soft" hadronic physics and pragmatic application of Pomeron phenomenology had been remarkably successful (see e.g. last issue of the Review of Particles Properties).

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[1]For the supercritical Pomeron $\alpha_P(0) - 1 = \Delta \ll 1$, $\Delta > 0$ is responsible for the growth of hadronic cross sections with energy.

That was the case before the appearance of CDF data on single diffractive dissociation [9,10] and recent results from HERA [11], which had shown that a popular model of supercritical Pomeron did not describe new experimental data. Obviously, the foundations of the Pomeron model require further theoretical study and the construction of newer, more general phenomenological framework, which would enable one to remove the discrepancy between the model predictions and the experiment. Of course, it is good that we have simple and compact form for representing a great variety of data for different hadronic processes, but it is certainly bad that power behaved total cross sections violate unitarity. Often and often encountered claim, that the model with power behaved total cross sections is valid in the non-asymptotic domain which has been explored up today, is not correct because supercritical Pomeron model is an asymptotic one by definition.

We suggested a new approach to the dynamical description of one-particle inclusive reactions [12]. The main point of our approach is that new fundamental three-body forces are responsible for the dynamics of particle production processes of inclusive type. Our consideration revealed several fundamental properties of one-particle inclusive cross-sections in the region of diffraction dissociation. In particular, it was shown that the slope of the diffraction cone in $p\bar{p}$ single diffraction dissociation was related to the effective radius of three-nucleon forces in the same way as the slope of the diffraction cone in elastic $p\bar{p}$ scattering was related to the effective radius of two-nucleon forces. It was also demonstrated that the effective radii of two- and three-nucleon forces, which were the characteristics of elastic and inelastic interactions of two nucleons, defined the structure of the total cross-sections in a simple and physically clear form. We made an attempt to check up the structure on its correspondence to the existing experimental data on proton-proton and proton-antiproton total cross sections. It is a remarkable fact, which is presented in this paper, that there is a very good correspondence.

First of all it should be elucidated what the three-body forces are. It will be made in the next sections.

II. THREE-BODY FORCES IN RELATIVISTIC QUANTUM THEORY

Using the LSZ or the Bogoljubov reduction formulae in quantum field theory [13], we can easily obtain the following cluster structure for $3 \rightarrow 3$ scattering amplitude (see Fig. 1)

$$\mathcal{F}_{123} = \mathcal{F}_{12} + \mathcal{F}_{23} + \mathcal{F}_{13} + \mathcal{F}_{123}^{C} \tag{1}$$

where $\mathcal{F}_{ij}, (i, j = 1, 2, 3)$ are $2 \rightarrow 2$ scattering amplitudes, $\mathcal{F}_{123}^{C}$ is called the connected part of the $3 \rightarrow 3$ scattering amplitude.
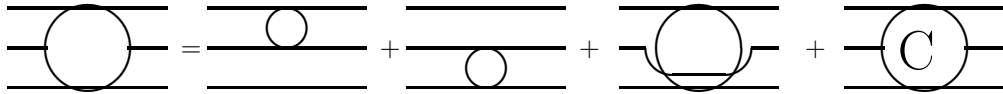

Figure 1: Cluster structure for $3 \rightarrow 3$ scattering amplitude.

In the framework of single-time formalism in quantum field theory [14] we construct the $3 \rightarrow 3$ off energy shell scattering amplitude $T_{123}(E)$ with the same (cluster) structure as (1)

$$T_{123}(E) = T_{12}(E) + T_{23}(E) + T_{13}(E) + T_{123}^{C}(E). \tag{2}$$

Following the tradition we'll call the kernel describing the interaction of three particles as the three particle interaction quasipotential. The three particle interaction quasipotential $V_{123}(E)$ is related to the off-shell $3 \rightarrow 3$ scattering amplitude $T_{123}(E)$ by the Lippmann-Schwinger type equation

$$T_{123}(E) = V_{123}(E) + V_{123}(E)G_0(E)T_{123}(E). \tag{3}$$

There exists the same transformation between two particle interaction quasipotentials V_{ij} and off energy shell $2 \rightarrow 2$ scattering amplitudes T_{ij}

$$T_{ij}(E) = V_{ij}(E) + V_{ij}(E)G_0(E)T_{ij}(E). \tag{4}$$

It can be shown that in the quantum field theory the three particle interaction quasipotential has the following structure [15]:

$$V_{123}(E) = V_{12}(E) + V_{23}(E) + V_{13}(E) + V_0(E). \tag{5}$$

The quantity $V_0(E)$ is called the three-body forces quasipotential. The $V_0(E)$ represents the defect of three particle interaction quasipotential over the sum of two particle interaction quasipotentials and describes the true three-body interactions. Three-body forces quasipotential is an inherent connected part of total three particle interaction quasipotential which can not be represented by the sum of pair interaction quasipotentials.

The three-body forces scattering amplitude is related to the three-body forces quasipotential by the equation

$$T_0(E) = V_0(E) + V_0(E)G_0(E)T_0(E). \tag{6}$$

It should be stressed that the three-body forces appear as a result of consistent consideration of three-body problem in the framework of local quantum field theory.

III. GLOBAL ANALYTICITY OF THE THREE-BODY FORCES

Let us introduce the following useful notations:

$$< p_1' p_2' p_3' |S - 1|p_1 p_2 p_3 >= 2\pi i \delta^4 (\sum_{i=1}^{3} p_i' - \sum_{j=1}^{3} p_j)\mathcal{F}_{123}(s; \hat{e}', \hat{e}), \tag{7}$$

$$s = (\sum_{i=1}^{3} p_i')^2 = (\sum_{j=1}^{3} p_j)^2.$$

$\hat{e}', \hat{e} \in S_5$ are two unit vectors on five-dimensional sphere describing the configuration of three-body system in initial and final states (before and after scattering).

We will denote the quantity T_0 restricted on the energy shell as

$$T_0 \mid_{on\ energy\ shell} = \mathcal{F}_0.$$

The unitarity condition for the quantity $\mathcal{F}_0$ with account for the introduced notations can be written in form [16,17]

$$Im\mathcal{F}_0(s; \hat{e}', \hat{e}) = \pi A_3(s) \int d\Omega_5(\hat{e}'')\mathcal{F}_0(s; \hat{e}', \hat{e}'') \overset{*}{\mathcal{F}_0} (s; \hat{e}, \hat{e}'') + H_0(s; \hat{e}', \hat{e}), \tag{8}$$

$$Im\mathcal{F}_0(s; \hat{e}', \hat{e}) = \frac{1}{2i} \left[\mathcal{F}_0(s; \hat{e}', \hat{e}) - \overset{*}{\mathcal{F}_0} (s; \hat{e}, \hat{e}') \right],$$

where

$$A_3(s) = \Gamma_3(s)/S_5,$$

$\Gamma_3(s)$ is the three-body phase-space volume, S_5 is the volume of unit five-dimensional sphere. H_0 defines the contribution of all inelastic channels emerging due to three-body forces.

Let us introduce a special notation for the scalar product of two unit vectors $\hat{e}'$ and $\hat{e}$

$$\cos \omega = \hat{e}' \cdot \hat{e}. \tag{9}$$

We will use the other notation for the three-body forces scattering amplitude as well

$$\mathcal{F}_0(s; \hat{e}', \hat{e}) = \mathcal{F}_0(s; \eta, \cos \omega),$$

where all other variables are denoted through η.

Now we are able to go to the formulation of our basic assumption on the analytical properties for the three-body forces scattering amplitude [16,17].

We will assume that for physical values of the variable s and fixed values of η the amplitude $\mathcal{F}_0(s; \eta, \cos \omega)$ is an analytical function of the variable $\cos \omega$ in the ellipse $E_0(s)$ with the semi-major axis

$$z_0(s) = 1 + \frac{M_0^2}{2s} \tag{10}$$

and for any $\cos\omega \in E_0(s)$ and physical values of η it is polynomially bounded in the variable s. M_0 is some constant having mass dimensionality.

Such analyticity of the three-body forces amplitude was called a global one. The global analyticity may be considered as a direct geometric generalization of the known analytical properties of two-body scattering amplitude strictly proved in the local quantum field theory [18–22].

At the same time the global analyticity results in the generalized asymptotic bounds. For example, the generalized asymptotic bound for $O(6)$-invariant three-body forces scattering amplitude looks like [16,17]

$$Im\,\mathcal{F}_0(s;...) \leq \text{Const}\,s^{3/2}\Big(\frac{\ln s/s_0'}{M_0}\Big)^5 = \text{Const}\,s^{3/2}R_0^5(s), \tag{11}$$

where $R_0(s)$ is the effective radius of the three-body forces introduced according to [22], where the effective radius of two-body forces has been defined

$$R_0(s) = \frac{\Lambda_0}{\Pi(s)} = \frac{r_0}{M_0}\ln\frac{s}{s_0'}, \quad \Pi(s) = \frac{\sqrt{s}}{2}, \quad s \to \infty, \tag{12}$$

r_0 is defined by the power of the amplitude $\mathcal{F}_0$ growth at high energies [17], M_0 defines the semi-major axis of the global analyticity ellipse (10), Λ_0 is the effective global orbital momentum, $\Pi(s)$ is the global momentum of three-body system, s_0' is a scale defining unitarity saturation of three-body forces.

It is well known that the Froissart asymptotic bound [23] can be experimentally verified because with the help of the optical theorem we can connect the imaginary part of $2 \to 2$ scattering amplitude with the experimentally measurable quantity which is the total cross section. So, if we want to have a possibility for the experimental verification of the generalized asymptotic bounds ($n \geq 3$) we have to establish a connection between the many-body forces scattering amplitudes and the experimentally measurable quantities. For this aim we have considered the problem of high energy particle scattering from deuteron and on this way we found the connection of the three-body forces scattering amplitude with the experimentally measurable quantity which is the total cross section for scattering from deuteron [24]. Moreover, the relation of the three-body forces scattering amplitude to one-particle inclusive cross sections has been established [25].

We shall briefly sketch now the basic results of our analysis of high energy particle scattering from deuteron.

IV. SCATTERING FROM DEUTERON

The problem of scattering from two-body bound states was treated in works [24,25] with the help of dynamic equations obtained on the basis of single-time formalism in QFT [15]. As has been shown in [24,25], the total cross section in scattering from deuteron can be expressed by the formula

$$\sigma_{hd}^{tot}(s) = \sigma_{hp}^{tot}(\hat{s}) + \sigma_{hn}^{tot}(\hat{s}) - \delta\sigma(s), \tag{13}$$

where $\sigma_{hd}, \sigma_{hp}, \sigma_{hn}$ are the total cross sections in scattering from deuteron, proton and neutron,

$$\delta\sigma(s) = \delta\sigma_G(s) + \delta\sigma_0(s), \tag{14}$$

$$\delta\sigma_G(s) = \frac{\sigma_{hp}^{tot}(\hat{s})\sigma_{hn}^{tot}(\hat{s})}{4\pi(R_d^2 + B_{hp}(\hat{s}) + B_{hn}(\hat{s}))} \equiv \frac{\sigma_{hp}^{tot}(\hat{s})\sigma_{hn}^{tot}(\hat{s})}{4\pi R_{eff}^2(s)}, \quad \hat{s} = \frac{s}{2}, \tag{15}$$

$B_{hN}(s)$ is the slope of the forward diffraction peak in the elastic scattering from nucleon, $1/R_d^2$ is defined by the deuteron relativistic formfactor

$$\frac{1}{R_d^2} \equiv \frac{q}{\pi}\int\frac{d\vec{\Delta}\Phi(\vec{\Delta})}{2\omega_h(\vec{q}+\vec{\Delta})}\delta\left[\omega_h(\vec{q}+\vec{\Delta}) - \omega_h(\vec{q})\right], \quad \frac{s}{2M_d} \cong q \cong \frac{\hat{s}}{2M_N}, \tag{16}$$

$\delta\sigma_G$ is the Glauber correction or shadow effect. The Glauber shadow correction originates from elastic rescatterings of an incident particle on the nucleons inside the deuteron.

The quantity $\delta\sigma_0$ represents the contribution of the three-body forces to the total cross section in scattering from deuteron. The physical reason for the appearance of this quantity is directly connected with the inelastic interactions of an incident particle with the nucleons of deuteron. Paper [25] provides for this quantity the following expression:

$$\delta\sigma_0(s) = -\frac{(2\pi)^3}{q} \int \frac{d\vec{\Delta}\Phi(\vec{\Delta})}{2E_p(\vec{\Delta}/2)2E_n(\vec{\Delta}/2)} Im\, R\Big(s; -\frac{\vec{\Delta}}{2}, \frac{\vec{\Delta}}{2}, \vec{q}; \frac{\vec{\Delta}}{2}, -\frac{\vec{\Delta}}{2}, \vec{q}\Big), \tag{17}$$

where q is the incident particle momentum in the lab system (rest frame of deuteron), $\Phi(\vec{\Delta})$ is the deuteron relativistic formfactor, normalized to unity at zero,

$$E_N(\vec{\Delta}) = \sqrt{\vec{\Delta}^2 + M_N^2} \quad N = p, n,$$

M_N is the nucleon mass. The function R is expressed via the amplitude of the three-body forces T_0 and the amplitudes of elastic scattering from the nucleons T_{hN} by the relation

$$R = T_0 + \sum_{N=p,n} (T_0 G_0 T_{hN} + T_{hN} G_0 T_0). \tag{18}$$

In [24] the contribution of three-body forces to the scattering amplitude from deuteron was related to the processes of multiparticle production in the inelastic interactions of the incident particle with the nucleons of deuteron. This was done with the help of the unitarity equation. The character of the energy dependence of $\delta\sigma_0$ was shown to be governed by the energy behaviour of the corresponding inclusive cross sections.

Here for simplicity let us consider the model where the imaginary part of the three-body forces scattering amplitude has the form

$$Im\, \mathcal{F}_0(s; \vec{p}_1, \vec{p}_2, \vec{p}_3; \vec{q}_1, \vec{q}_2, \vec{q}_3) = f_0(s) \exp\Big\{ -\frac{R_0^2(s)}{4} \sum_{i=1}^{3} (\vec{p}_i - \vec{q}_i)^2 \Big\}, \tag{19}$$

where $f_0(s)$, $R_0(s)$ are free parameters which in general may depend on the total energy of three-body interaction. Note that the quantity $f_0(s)$ has the dimensionality $[R^2]$. The model assumption (19) is not significant for our main conclusions but allows one to make some calculations exactly in a closed form.

In case of unitarity saturation of the three-body forces, we have from the generalized asymptotic theorems

$$f_0(s) \sim \text{Const}\, s^{3/2} \Big(\frac{\ln s/s_0'}{M_0}\Big)^5 = \text{Const}\, s^{3/2} R_0^5(s), \tag{20}$$

$$R_0(s) = \frac{r_0}{M_0} \ln s/s_0' \quad s \to \infty. \tag{21}$$

In the model all the integrals can be calculated in the analytical form. As a result we obtain for the quantity $\delta\sigma_0$ [25]

$$\delta\sigma_0(s) = \frac{(2\pi)^6 f_0(s)}{sM_N} \Big\{ \frac{\sigma_{hN}(s/2)}{2\pi[B_{hN}(s/2) + R_0^2(s) - R_0^4(s)/4(R_0^2(s) + R_d^2)]} - 1 \Big\} \frac{1}{[2\pi(R_d^2 + R_0^2(s))]^{3/2}}. \tag{22}$$

If the condition

$$R_0^2(s) \simeq B_{hN}(s/2) \ll R_d^2 \tag{23}$$

is realized, then we obtain from expression (22)

$$\delta\sigma_0(s) = (2\pi)^{9/2} \frac{f_0(s)\chi(s)}{sM_N R_d^3}, \tag{24}$$

where

$$\chi(s) = \frac{\sigma_{hN}^{tot}(s/2)}{2\pi[B_{hN}(s/2) + R_0^2(s)]} - 1, \tag{25}$$

and we suppose that asymptotically

$$B_{hp} = B_{hn} \equiv B_{hN}, \quad \sigma_{hp}^{tot} = \sigma_{hn}^{tot} \equiv \sigma_{hN}^{tot}.$$

It should be noted that (23) is justified by the physical conditions which are realized in the problem of high-energy particle scattering from deuteron at the recently available energies. Of course, there is some energy where (23) can no longer be true and this situation can be considered as well. We can calculate this energy. From an experiment it is known that $R_d^2 \cong 100\,GeV^{-2}$. Then we can easily calculate (see Eqs. (47,48) later) that $2B_{hN}(s_1) = R_d^2$ at $\sqrt{s_1} = 1515.92\,TeV$. In the same way (see Eq. (50) later) we can find that $R_0^2(s_2) = R_d^2$ at $\sqrt{s_2} = 77.43\,PeV$. So, both s_1 and s_2 are extremely high energies.

In the 50th Bogoljubov proposed the idea to describe stable compound systems by local fields. Bogoljubov's idea has brilliantly been realized by Zimmermann in his famous paper [26]. Zimmermann's construction for local deuteron field looks like

$$B_d(X) = \lim_{x \to 0} \frac{T\left(\Phi_p(X + \frac{1}{2}x)\Phi_n(X - \frac{1}{2}x)\right)}{< 0|T\left(\Phi_p(\frac{1}{2}x)\Phi_n(-\frac{1}{2}x)\right)|d>}. \tag{26}$$

It has been proved [26] that $B_d(X)$ satisfies microlocal causality. Moreover, the asymptotic deuteron fields constructed with the Yang–Feldman procedure fulfil the commutation relations of Fock representation. The hadron–deuteron scattering amplitude can be presented by LSZ reduction formula in the form

$$< \vec{P}_d\vec{p}_h|S - 1|\vec{Q}_d\vec{q}_h >= \iint dX\,dx \iint dY\,dy \overset{*}{f}_{M_d;\vec{P}_d}(X)\overset{*}{f}_{m_h;\vec{p}_h}(x)\times$$

$$\overset{\to M_d}{K_X}\overset{\to m_h}{K_x}< 0|T\left(B_d(X)\Phi_h(x)B_d(Y)\Phi_h(y)\right)|0 > \overset{\leftarrow M_d}{K_Y}\overset{\leftarrow m_h}{K_y}f_{M_d;\vec{Q}_d}(Y)f_{m_h;\vec{q}_h}(y) \tag{27}$$

where $K_x^m \equiv \Box_x + m^2$ is Klein-Gordon-Fock differential operator,

$$f_{m;\vec{p}}(x) = (2\pi)^{-3/2}exp(-ipx), \quad p^0 = E(\vec{p}, m) = \sqrt{\vec{p}^2 + m^2}.$$

Of course, the construction of local interpolating Heisenberg fields is not a unique procedure. There are equivalence classes of different fields (Borchers's classes [27]), which have the same asymptotic fields and give rise to the same S-matrix. Anyway Zimmermann's construction allows us to use the local quantum field theory Causality-Spectrality–Analyticity-Unitarity (CS–AU) machine [13] and prove the Froissart theorem for hadron–deuteron elastic scattering amplitude as well and, as a consequence, obtain the Froissart bound for total cross section in hadron–deuteron interaction.

First two terms in Eq. (13) fulfil the Froissart bound. The quantity $\delta\sigma_G$ meets a stronger bound than the Froissart one because we know that

$$\sigma_{hN}^{tot}(s, s_0) \sim \ln^2(s/s_0) \Longrightarrow B_{hN}(s, s_0) \sim \ln^2(s/s_0),$$

and, therefore, we have from Eq. (15)[2]

$$\delta\sigma_G(s) < 2\sigma_{hN}^{el}(\hat{s}), \quad \left(\sigma_{hN}^{el}(s) = \frac{\sigma_{hN}^{tot}(s)^2}{16\pi B_{hN}(s)}, \quad \rho_{el} = 0\right).$$

Obviously, from Eq. (15) it follows that the bound $\delta\sigma_G(s) < 2\sigma_{hN}^{el}(\hat{s})$ is true at any relation between B_{hN} and R_d^2. But if the condition (23) is realized we have a more stronger bound

$$\delta\sigma_G(s) < 2\epsilon(\hat{s})\sigma_{hN}^{el}(\hat{s}), \quad \epsilon(\hat{s}) = 2B_{hN}(\hat{s})/R_d^2 \ll 1.$$

From expression (24) for the correction $\delta\sigma_0$ in the case of unitarity saturation of the three-body forces with the energy dependence given by Eq. (20) then it follows that

$$\delta\sigma_0(s) < C_0\,ln^2s \quad (C_0 < C_{hN}^F = \pi/m_\pi^2) \quad s \to \infty$$

if and only if the asymptotic bound

$$\chi(s) < \frac{C}{\sqrt{s}\ln^3 s}, \quad s \to \infty. \tag{28}$$

is valid.

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[2]The bound $\delta\sigma_G(s) < 2\sigma_{hN}^{el}(\hat{s})$ is also true when $\rho_{el} \neq 0$.

V. GLOBAL STRUCTURE OF HADRONIC TOTAL CROSS SECTIONS

Let's rewrite the equation for $\chi(s)$

$$\chi(s) = \frac{\sigma_{hN}^{tot}(s/2)}{2\pi[B_{hN}(s/2) + R_0^2(s)]} - 1$$

in the form

$$\sigma_{hN}^{tot}(s) = 2\pi \left[B_{hN}(s) + R_0^2(2s)\right](1 + \chi). \tag{29}$$

From the Froissart and generalized asymptotic bounds we have

$$\chi(s) = O\left(\frac{1}{\sqrt{s}\ln^3 s}\right), \quad s \to \infty.$$

We also know that [20]

$$\sigma_{hN}^{tot}(s, s_0) \sim \ln^2(s/s_0) \implies B_{hN}(s, s_0) \sim \ln^2(s/s_0), \tag{30}$$

and Eq. (29) gives

$$R_0^2(2s, s_0') \sim \ln^2(2s/s_0') \sim \ln^2(s/s_0), \quad s \to \infty.$$

Therefore we come to the following asymptotic consistency condition

$$\boxed{s_0' = 2s_0}. \tag{31}$$

The asymptotic consistency condition tells us that we have not any new scale. The scale defining unitarity saturation of three-body forces is unambiguously expressed by the scale which defines unitarity saturation of two-body forces. In that case we have

$$R_0^2(2s, s_0') = R_0^2(s, s_0)$$

and

$$\boxed{\sigma_{hN}^{tot}(s) = 2\pi \left[B_{hN}(s) + R_0^2(s)\right](1 + \chi(s))} \tag{32}$$

with a common scale s_0.

Reminding the relation between the effective radius of two-body forces and the slope of diffraction cone in elastic scattering

$$B_{hN}(s) = \frac{1}{2}R_{hN}^2(s) \tag{33}$$

we obtain

$$\sigma_{hN}^{tot}(s) = \pi R_{hN}^2(s) + 2\pi R_0^2(s), \quad s \to \infty. \tag{34}$$

Equations (32) and (34) define new nontrivial structure of hadronic total cross section. It should be emphasized that the coefficients staying in the R.H.S. of Eq. (34) in front of effective radii of two- and three-body forces are strongly fixed.

It is useful to compare the new structure of total hadronic cross section with the known one. We have from unitarity

$$\sigma_{hN}^{tot}(s) = \sigma_{hN}^{el}(s) + \sigma_{hN}^{inel}(s). \tag{35}$$

If we put

$$\sigma_{hN}^{el}(s) = \pi R_{hN}^{el\,2}(s), \quad \sigma_{hN}^{inel}(s) = 2\pi R_{hN}^{inel\,2}(s), \tag{36}$$

then we come to the similar formula

$$\sigma_{hN}^{tot}(s) = \pi R_{hN}^{el\ 2}(s) + 2\pi R_{hN}^{inel\ 2}(s). \tag{37}$$

But it should be borne in mind

$$R_{hN}^2(s) \neq R_{hN}^{el\ 2}(s), \quad R_0^2(s) \neq R_{hN}^{inel\ 2}(s). \tag{38}$$

In fact we have

$$\sigma_{hN}^{el}(s) = \frac{\sigma_{hN}^{tot\ 2}(s)}{16\pi B_{hN}(s)} = \frac{\sigma_{hN}^{tot\ 2}(s)}{8\pi R_{hN}^2(s)}, \tag{39}$$

$$\sigma_{hN}^{inel}(s) = \sigma_{hN}^{tot}(s) \left[1 - \frac{\sigma_{hN}^{tot}(s)}{8\pi R_{hN}^2(s)} \right]. \tag{40}$$

Of course, Eqs. (36) are the definitions of R_{hN}^{el} and R_{hN}^{inel}. The definition of R_{hN}^{el} corresponds to our classical imagination, the definition of R_{hN}^{inel} corresponds to our knowledge of quantum mechanical problem for scattering from a black disk. Let us suppose that

$$\sigma_{hN}^{tot}(s_m) \cong \pi R_{hN}^2(s_m), \quad s_m \in \mathcal{M}, \quad \left(R_0^2(s_m) \ll R_{hN}^2(s_m) \right), \tag{41}$$

then we obtain

$$\sigma_{hN}^{el}(s_m) = \frac{1}{8}\pi R_{hN}^2(s_m), \quad \sigma_{hN}^{inel}(s_m) = \frac{7}{8}\pi R_{hN}^2(s_m). \tag{42}$$

This simple example shows that the new structure of total hadronic cross sections is quite different from that given by Eq. (35). The reason is that structure (32) is of dynamical origin. We have mentioned above that the coefficients, staying in R.H.S. of Eq. (34) in front of effective radii of two- and three-body forces, are strongly fixed. In fact, we found here the answer to the old question: Why the constant $(\pi/m_\pi^2 \approx 60\,mb)$ staying in the Froissart bound is too large in the light of existing experimental data. The constant in R.H.S. of Eq. (34), staying in front of the effective radius of hadron-hadron interaction, is 4 times smaller than the constant in the Froissart bound. But this is too small to correspond to the experimental data. The second term in R.H.S. of Eq. (34) fills an emerged gap. Besides from the Froissart bound

$$\sigma_{hN}^{tot}(s) \leq 4\pi R_{hN}^2(s)$$

we obtain the bound for the effective radius of three-body forces

$$R_0^2(s) \leq \frac{3}{2} R_{hN}^2(s). \tag{43}$$

It is a remarkable fact that the quantity R_0^2, which has the clear physical interpretation, at the same time, is related to the experimentally measurable quantity which the total cross section is. This important circumstance gives rise to the nontrivial consequences [28].

We made an attempt to check up the structure (32) on its correspondence to the existing experimental data. Our results are presented below.

At the first step, we made a weighted fit to the experimental data on the proton-antiproton total cross sections in the range $\sqrt{s} > 10\,GeV$. The data were fitted with the function of the form predicted by the Froissart bound in the spirit of our approach[3]

$$\sigma_{asmpt}^{tot} = a_0 + a_2 \ln^2(\sqrt{s}/\sqrt{s_0}) \tag{44}$$

where $a_0, a_2, \sqrt{s_0}$ are free parameters. We accounted for the experimental errors δx_i (statistical and systematic errors added in quadrature) by fitting to the experimental points with the weight $w_i = 1/(\delta x_i)^2$. Our fit yielded

$$a_0 = (42.0479 \pm 0.1086)mb, \quad a_2 = (1.7548 \pm 0.0828)mb, \tag{45}$$

$$\sqrt{s_0} = (20.74 \pm 1.21)GeV. \tag{46}$$

The fit result is shown in Fig. 2.

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[3]Recently, from a careful analysis of the experimental data and comparative study of the known characteristic parameterizations, Bueno and Velasco have shown that statistically a "Froissart-like" type parameterization for proton-proton and proton-antiproton total cross sections is strongly favoured [29].
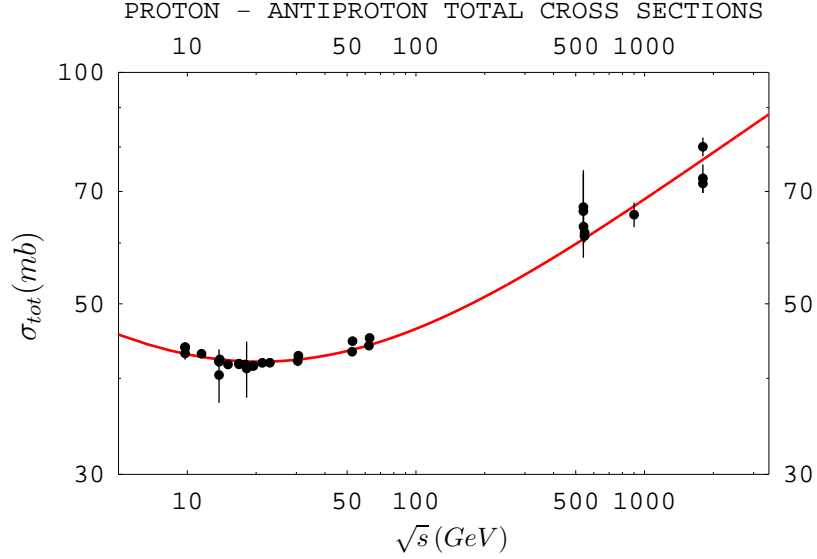
Figure 2: The total proton-antiproton cross sections versus $\sqrt{s}$ compared with formula (44). Solid line represents our fit to the data. Statistical and systematic errors added in quadrature.

After that we made a weighted fit to the experimental data on the slope of diffraction cone in elastic $p\bar{p}$ scattering. The experimental points and the references, where they have been extracted from, are listed in [30]. The fitted function of the form

$$B = b_0 + b_2 \ln^2(\sqrt{s}/20.74), \qquad (47)$$

which is also suggested by the asymptotic theorems of local quantum field theory, has been used. The value $\sqrt{s_0}$ was fixed by (46) from the fit to the $p\bar{p}$ total cross sections data. Our fit yielded

$$b_0 = (11.92 \pm 0.15)GeV^{-2}, \quad b_2 = (0.3036 \pm 0.0185)GeV^{-2}. \qquad (48)$$
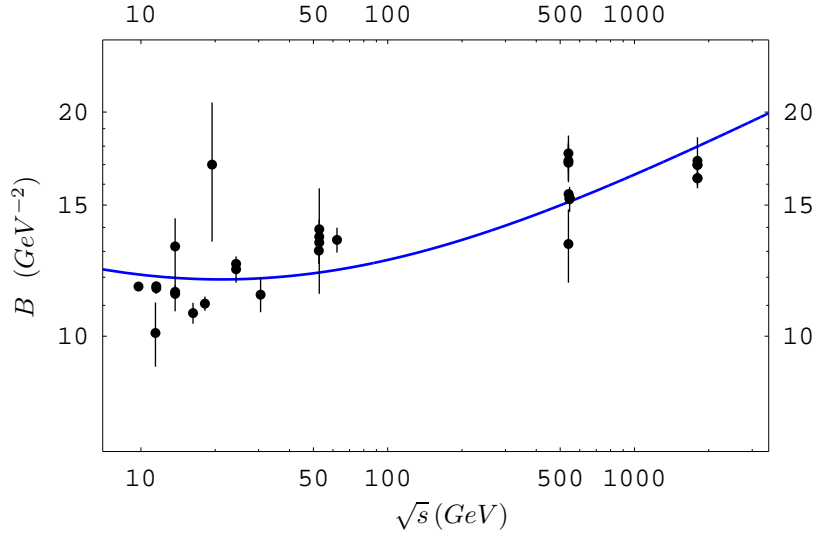
The fitting curve is shown in Fig. 3.

Figure 3: Slope B of diffraction cone in $p\bar{p}$ elastic scattering. Solid line represents our fit to the data.

At the final stage we build a global (weighted) fit to the all data on proton-antiproton total cross sections in a whole range of energies available up today. The global fit was made with the function of the form

$$\sigma_{p\bar{p}}^{tot}(s) = \sigma_{asmpt}^{tot}(s)\left[1 + \frac{c}{\sqrt{s - 4m_N^2}R_0^3(s)}\left(1 + \frac{d_1}{\sqrt{s}} + \frac{d_2}{s} + \frac{d_3}{s^{3/2}}\right)\right] \qquad (49)$$

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where m_N is proton (nucleon) mass,

$$R_0^2(s) = \left[0.40874044\,\sigma_{asmpt}^{tot}(s)(mb) - B(s) \right] (GeV^{-2}), \tag{50}$$

$$\sigma_{asmpt}^{tot}(s) = 42.0479 + 1.7548 \ln^2(\sqrt{s}/20.74), \tag{51}$$

$$B(s) = 11.92 + 0.3036 \ln^2(\sqrt{s}/20.74), \tag{52}$$

c, d_1, d_2, d_3 are free parameters. Function (49) corresponds to the structure given by Eq. (32).

In fact we have for the function $\chi(s)$ in the R.H.S. of Eq. (32) the theoretical expression in the form[4]

$$\chi(s) = \frac{C}{\kappa(s)R_0^3(s)}, \tag{53}$$

where

$$\kappa^4(s) = \frac{1}{2\pi} \int_a^b dx \sqrt{(x^2 - a^2)(b^2 - x^2)[(a+b)^2 - x^2]}, \quad a = 2m_N, \quad b = \sqrt{2s + m_N^2} - m_N. \tag{54}$$

It can be proved that $\kappa(s)$ has the following asymptotics[5]

$$\kappa(s) \sim \sqrt{s}, \quad s \to \infty; \qquad \kappa(s) \sim \sqrt{s - 4m_N^2}, \quad s \to 4m_N^2.$$

We used at the moment the simplest function staying in the R.H.S. of Eq. (49) which described these two asymptotics, but at this price we introduced the preasymptotic terms needed to describe the region of middle (intermediate) energies.

Our fit yielded

$$d_1 = (-12.12 \pm 1.023)GeV, \quad d_2 = (89.98 \pm 15.67)GeV^2,$$

$$d_3 = (-110.51 \pm 21.60)GeV^3, \quad c = (6.655 \pm 1.834)GeV^{-2}. \tag{55}$$

The fitting curve is shown in Fig. 4.

PROTON – ANTIPROTON TOTAL CROSS SECTIONS

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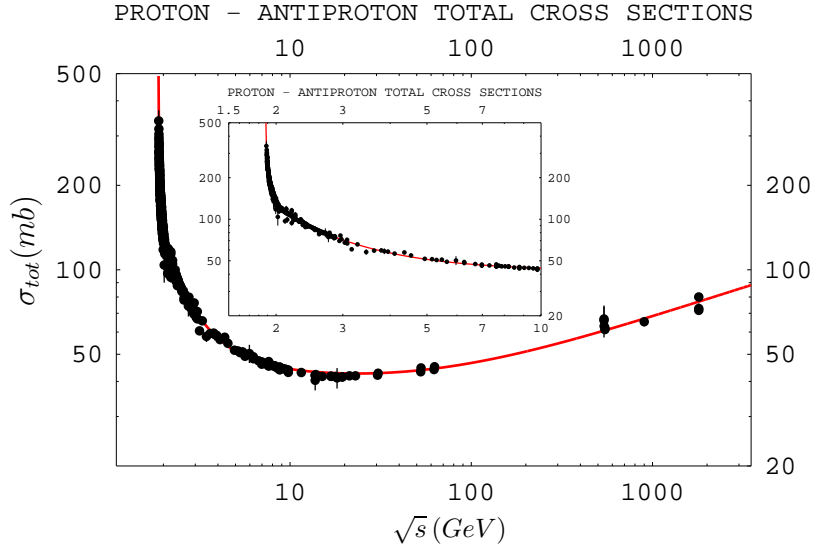
^{4}(53) follows from (24) if $\delta\sigma_0(s) \sim CR_0^2(s)$ and we assume that $R_0^2(s) \ll R_d^2$ which is justified up to extremely high energies.

^{5}Integral in R.H.S. of Eq. (54) can be expressed in terms of the Appell function.

Figure 4: The total proton-antiproton cross sections versus $\sqrt{s}$ compared with formula (49). Solid line represents our fit to the data.

The experimental data on proton-proton total cross sections display a more complex structure at low energies than the proton-antiproton ones. To describe this complex structure we, of course, have to modify formula (49) without destroying the general structure given by Eq. (32). Modified formula looks like

$$\sigma_{pp}^{tot}(s) = \sigma_{asmpt}^{tot}(s) \left[1 + \left(\frac{c_1}{\sqrt{s - 4m_N^2} R_0^3(s)} - \frac{c_2}{\sqrt{s - s_{thr}} R_0^3(s)} \right) (1 + d(s)) + Resn(s) \right], \qquad (56)$$

where $\sigma_{asmpt}^{tot}(s)$ is the same as in the proton-antiproton case (Eq. (51)) and

$$d(s) = \sum_{k=1}^{8} \frac{d_k}{s^{k/2}}, \quad Resn(s) = \sum_{i=1}^{8} \frac{C_R^i s_R^i \Gamma_R^i{}^2}{\sqrt{s(s - 4m_N^2)}[(s - s_R^i)^2 + s_R^i \Gamma_R^i{}^2]}. \qquad (57)$$

Compared to Eq. (49) we introduced here an additional term $Resn(s)$ describing diproton resonances which have been extracted from [31,32]. The positions of resonances and their widths are listed in Table I.

Table I: Diproton resonances extracted from papers [31,32].

$m_R(MeV)$	$\Gamma_R(MeV)$	$C_R(GeV^2)$
1937 ± 2	7 ± 2	0.0722 ± 0.0235
1955 ± 2	9 ± 4	0.1942 ± 0.0292
1965 ± 2	6 ± 2	0.1344 ± 0.0117
1980 ± 2	9 ± 2	0.3640 ± 0.0654
2008 ± 3	4 ± 2	0.3234 ± 0.0212
2106 ± 2	11 ± 5	-0.2958 ± 0.0342
2238 ± 3	22 ± 8	0.4951 ± 0.0559
2282 ± 4	24 ± 9	0.0823 ± 0.0319

The $c_1, c_2, s_{thr}, d_i, C_R^i (i = 1, ..., 8)$ were considered as free fit parameters. Fitted parameters obtained by fit are listed below (see C_R^i in Table I)

$$c_1 = (192.85 \pm 1.68) GeV^{-2}, \quad c_2 = (186.02 \pm 1.67) GeV^{-2},$$

$$s_{thr} = (3.5283 \pm 0.0052) GeV^2,$$

$$d_1 = (-2.197 \pm 1.134) 10^2 GeV, \quad d_2 = (4.697 \pm 2.537) 10^3 GeV^2,$$

$$d_3 = (-4.825 \pm 2.674) 10^4 GeV^3, \quad d_4 = (28.23 \pm 15.99) 10^4 GeV^4,$$

$$d_5 = (-98.81 \pm 57.06) 10^4 GeV^5, \quad d_6 = (204.5 \pm 120.2) 10^4 GeV^6,$$

$$d_7 = (-230.2 \pm 137.3) 10^4 GeV^7, \quad d_8 = (108.26 \pm 65.44) 10^4 GeV^8. \qquad (58)$$

The fitting curve is shown in Fig. 5. It should be pointed out that our fit revealed the fact that the resonance with the mass $m_R = 2106\, MeV$ should be odd parity. Our fit indicates that this resonance is strongly confirmed by the set of experimental data on proton-proton total cross sections. That is why a further study of diproton resonances is very desirable.
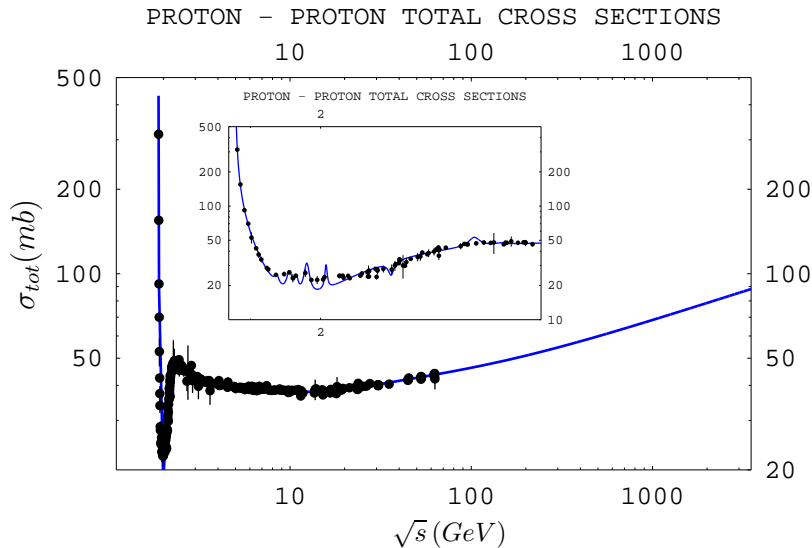
Figure 5: The total proton-proton cross sections versus $\sqrt{s}$ compared with formula (56). Solid line represents our fit to the data. Statistical and systematic errors added in quadrature.

Figures 4, 5 display a very good correspondence of theoretical formula (32) to the existing experimental data on proton-proton and proton-antiproton total cross sections.

VI. CONCLUSION

In conclusion we'd like to emphasize the following attractive features of formula (32). This formula represents hadronic total cross section in a factorized form. One factor describes high energy asymptotics of total cross section and it has the universal energy dependence predicted by the Froissart theorem. The other factor is responsible for the behaviour of total cross section at low energies and this factor has also the universal asymptotics at elastic threshold. It is a remarkable fact that the low energy asymptotics of total cross section at elastic threshold is dictated by high energy asymptotics of three-body (three-nucleon in that case) forces. This means that we undoubtedly faced very deep physical phenomena here. The appearance of new threshold $s_{thr} = 3.5283\,GeV^2$ in the proton-proton channel, which is near the elastic threshold, is nontrivial fact too. It's clear that the difference of two identical terms with different thresholds in R.H.S. of Eq. (56) is a tail of the crossing symmetry which is not actually taken into account in our consideration. What physical entity does this new threshold corresponds to? This interesting question is still open.

Anyway we have established that simple theoretical formula (32) described the global structure of pp and $p\bar{p}$ total cross sections in the whole range of energies available up today. We have shown that this formula follows from the generalized asymptotic theorems a là Froissart. It is very nice that the understanding of "soft" physics based on general principles of QFT, such as analyticity and unitarity, together with dynamic apparatus of single-time formalism in QFT, corresponds so fine to the experimentally observable picture.

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